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# DYNAMIC STABILITY OF THREE-LAYER PLATES WITH REGARD TO TRANSVERSE DEFORMATIONS IN A FILLER

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The equations of stability of three-layer plates taking into account transverse deformations in a filler obtained in [1] make it possible to study local loss of stability of load-carrying layers that can occur at lesser critical load than that required for general loss of stability. The problem on dynamic loss of stability of three-layer plates and rods is more interesting from the practical point of view. As three-layer plates and rods are elements of structures exploited under dynamic in-plane loads, it is necessary to take into account the possibility of the two above mentioned forms of loss of stability.

In this paper the equations and boundary conditions of the problem of dynamic stability of three-layer plates are obtained by a variational method. The applied method allows one to establish a complete correspondence between the force and kinematic factors and to derive correct boundary conditions.

We shall use the notation in [1]. The coordinate plane $x_1x_2$ is aligned with the middle surface of the filler. Let us denote the sizes of the plate along the edges by $l_1$, $l_2$; $h_1$, $h_2$, $h_3 = 2c$ are, respectively, the thicknesses of the load-carrying layers and the filler. Kirchhoff's hypothesis is used for the load-carrying layers, and the filler is regarded as a transversally-isotropic elastic body with the plane of isotropy parallel to its middle one.

Displacements $w^3$ along the thickness of the filler are linear

$$w^3 = \frac{1}{2}(w^1 + w^2) + \frac{1}{2}x_3c^{-1}(w^1 - w^2).$$

Tangential displacements in the layers are represented as

$$u_i^1 = u_i + ca_i - \frac{c}{4}(w^1 - w^2)_{,i} - (x_3 - c)w^1{}_{,i} \quad (c \leqslant x_3 \leqslant c + h_1),$$

$$u_i^2 = u_i - ca_i - \frac{c}{4}(w^1 - w^2)_{,i} - (x_3 + c)w^2{}_{,i} \quad (-h_2 - c \leqslant x_3 \leqslant -c),$$

$$u_i^3 = u_i + x_3a_i - \frac{x_3^2}{4c}(w^1 - w^2)_{,i} \quad (-c \leqslant x_3 \leqslant c) \quad (i = 1,\, 2),$$

where $u_i$ and $2ca_i$ are the displacements of the points of the middle surface and the absolute shears of the boundary planes of the filler along the $x_i$ axis; the index $i$ after the comma denotes differentiation with respect to coordinate $x_i$.

Hereafter the following two functions are used:

$$w = \frac{1}{2}(w^1 + w^2), \quad v = \frac{1}{2}(w^1 - w^2).$$

Strains are determined in the standard manner. Stresses are related to strains by Hooke's law. Their full expressions are presented in [1].

We derive the equations of dynamic stability of a three-layer plate using the Hamilton–Ostrogradskii principle [2]:

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$$\int_{t_1}^{t_2} \delta L\, dt = 0.$$

Here $t_1$, $t_2$ are the fixed initial and final instants of time; $\delta$ means the variation; $L$ is the Lagrangian;

$$\delta L = \delta K - \delta U + \delta A \tag{1}$$

($A$ is the work of the external forces; $U$ is the strain energy of warping; $K$ is the kinetic energy.) Variations of the strain energy of warping and the work of external contour forces are presented in [1]:

$$\delta U = \int_0^{l_1}\int_0^{l_2} \Big[\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{3} \int_{(h_k)} \sigma_{ij}^k \delta\varepsilon_{ij}^k\, dx_3 + \sum_{i=1}^{3} \int_{(h_3)} \sigma_{i3}^3 \delta\varepsilon_{i3}^3\, dx_3\Big]\, dx_1\, dx_2,$$
$$\delta A = \frac{1}{2}\int_0^{l_1}\int_0^{l_2} \Big[\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{3} \frac{1}{h_k}\int_{(h_k)} T_{ij}^k\Big(\frac{\partial w^k}{\partial x_i}\delta\frac{\partial w^k}{\partial x_j} + \frac{\partial w^k}{\partial x_j}\delta\frac{\partial w^k}{\partial x_i}\Big)\, dx_3\Big]\, dx_1\, dx_2. \tag{2}$$

Let us write the variation of kinetic energy of the plate in line with [2] in the form of

$$\delta K = -\int_0^{l_1}\int_0^{l_2} \Big[\sum_{i=1}^{2}\sum_{k=1}^{3} \int_{(h_k)} \rho_k \frac{\partial^2 u_i^k}{\partial t^2}\delta u_i^k\, dx_3 + \sum_{k=1}^{3} \int_{(h_k)} \rho_k \frac{\partial^2 w^k}{\partial t^2}\delta w^k\, dx_3\Big]\, dx_1\, dx_2. \tag{3}$$

In the expression (2) $T_{ij}^k$ are the contour forces acting along the contour of the $k$th layer; in the subcritical state, in order for the moments to be absent, these forces should be proportional to the rigidities $\gamma_k$, i.e., $T_{ij}^k = \gamma_k T_{ij}$ ($T_{ij}$ is the resultant of the external forces applied to the layers).

Let us introduce the notation: $h = h_1 + h_2 + h_3$, $t_k = h_k/h$, $E = E_1 t_1 + E_2 t_2 + E_3 t_3$, $\gamma_k = E_k h_k/E$, $\rho = h^{-1}\sum_{k=1}^{3} \rho_k h_k$, $\bar\gamma_k = \rho_k h_k (\rho h)^{-1}$. Here $E_k$ and $\rho_k$ are the Young's modulus and densities of the layers ($k = 1, 2, 3$). Evidently, we have the equalities $\sum_k \gamma_k = 1$, $\sum_k t_k = 1$, $\sum_k \bar\gamma_k = 1$ ($\gamma_k$ and $\bar\gamma_k$ are, respectively, the dimensionless rigidities and densities of the layers respectively). Then

$$\delta A - \delta U = \int_0^{l_1}\int_0^{l_2}\Big\{\sum_{i=1}^{2}\Big[(N_{1i,1} + N_{2i,2})\delta u_i + (H_{1i,1} + H_{2i,2} - Q_i^3)\delta a_i\Big] + \sum_{i=1}^{2}\Big[\sum_{j=1}^{2}(M_{ij,ij} - T_{ij}R_{,ij}) + Q_{i,i}^3\Big]\delta w$$
$$+ \Big[\sum_{i=1}^{2}\sum_{j=1}^{2}(L_{ij,ij} - T_{ij}S_{,ij}) - \frac{1}{c}Q_3^3\Big]\delta v\Big\}\, dx_1\, dx_2 - \int_0^{l_1}\Big[\sum_{i=1}^{2}(N_{i2}\delta u_i + H_{i2}\delta a_i) + (M_{22,2} + 2M_{12,1}$$
$$+ Q_2^3 - T_{22}R_{,2} - T_{12}R_{,1})\delta w + (L_{22,2} + 2L_{12,1} - T_{22}S_{,2} - T_{12}S_{,1})\delta v - M_{22}\delta w_{,2} - L_{22}\delta v_{,2}\Big]_0^{l_2} dx_1$$
$$- \int_0^{l_2}\Big[\sum_{i=1}^{2}(N_{i1}\delta u_i + H_{i1}\delta a_i) + (M_{11,1} + 2M_{12,2} + Q_1^3 - T_{11}R_{,1} - T_{12}R_{,2})\delta w \tag{4}$$
$$+ (L_{11,1} + 2L_{12,2} - T_{11}S_{,1} - T_{12}S_{,2})\delta v - M_{11}\delta w_{,1} - L_{11}\delta v_{,1}\Big]_0^{l_1} dx_2 + 2[M_{12}\delta w + L_{12}\delta v]_{0\,0}^{l_1 l_2},$$
$$R = w + (\gamma_1 - \gamma_2)v, \quad S = (\gamma_1 - \gamma_2)w + (\gamma_1 + \gamma_2 + \frac{1}{3}\gamma_3)v.$$

Specific efforts are introduced into these equations [1]

$$N_{ij} = N_{ij}^1 + N_{ij}^2 + N_{ij}^3, \quad M_{ij} = M_{ij}^1 + M_{ij}^2,$$

$$H_{ij} = M_{ij}^3 + c(N_{ij}^1 - N_{ij}^2), \quad L_{ij} = M_{ij}^1 - M_{ij}^2 + \frac{1}{2}c(N_{ij}^1 + N_{ij}^2) + G_{ij}^3,$$

$$N_{ij}^k = \int\limits_{(h_k)} \sigma_{ij}^k \, dx_3, \quad M_{ij}^1 = \int\limits_{(h_1)} \sigma_{ij}^1 (x_3 - c) \, dx_3, \quad M_{ij}^2 = \int\limits_{(h_2)} \sigma_{ij}^2 (x_3 + c) \, dx_3,$$

$$M_{ij}^3 = \int\limits_{(h_3)} \sigma_{ij}^3 x_3 \, dx_3, \quad G_{ij}^3 = \frac{1}{2}c^{-1} \int\limits_{(h_3)} \sigma_{ij}^3 x_3^2 \, dx_3, \quad Q_i^3 = \int\limits_{(h_3)} \sigma_{i3}^3 \, dx_3.$$

Let us write the variation of kinetic energy $\delta K$ defined by (3) as

$$\delta K = -\frac{\partial^2}{\partial t^2} \int\limits_0^{l_1}\int\limits_0^{l_2} \Big\{ \sum_{i=1}^2 \rho h [u_i + \frac{h}{2}\bar{c}_{12} a_i - \frac{h}{2}\bar{c}_{13} w_{,i} - \frac{h}{2}\bar{c}_{14} v_{,i}] \delta u_i + \sum_{i=1}^2 \rho h [\bar{c}_{12} u_i + \frac{h}{2}\bar{c}_{22} a_i - \frac{h}{2}\bar{c}_{23} w_{,i} - \frac{h}{2}\bar{c}_{24} v_{,i}] \delta a_i$$

$$+ \frac{\rho h^2}{2} \sum_{i=1}^2 [\bar{c}_{13} u_{i,i} + \frac{h}{2}\bar{c}_{23} a_{i,i} - \frac{h}{2}\bar{c}_{33} w_{,ii} - \frac{h}{2}\bar{c}_{34} v_{,ii} + h^{-1} w + (t_3 h)^{-1}(\bar{c}_{12} + \bar{c}_{13}) v] \delta w$$

$$+ \frac{\rho h^2}{2} \sum_{i=1}^2 [\bar{c}_{14} u_{i,i} + \frac{h}{2}\bar{c}_{24} a_{i,i} - \frac{h}{2}\bar{c}_{34} w_{,ii} - \frac{h}{2}\bar{c}_{44} v_{,ii} + (t_3 h)^{-1}(\bar{c}_{12} + \bar{c}_{13}) w + \frac{1}{3} t_3^{-2} h^{-1} \bar{c}_{55} v] \delta v \Big\} \, dx_1 \, dx_2$$

$$+ \frac{\rho h^2}{2} \frac{\partial^2}{\partial t^2} \int\limits_0^{l_2} [(\bar{c}_{13} u_1 + \frac{h}{2}\bar{c}_{23} a_1 - \frac{h}{2}\bar{c}_{33} w_{,1} - \frac{h}{2}\bar{c}_{34} v_{,1}) \delta w + (\bar{c}_{14} u_1 + \frac{h}{2}\bar{c}_{24} a_1 - \frac{h}{2}\bar{c}_{34} w_{,1} - \frac{h}{2}\bar{c}_{44} v_{,1}) \delta v]_0^{l_1} \, dx_2$$

$$+ \frac{\rho h^2}{2} \frac{\partial^2}{\partial t^2} \int\limits_0^{l_1} [\bar{c}_{13} u_2 + \frac{h}{2}\bar{c}_{23} a_2 - \frac{h}{2}\bar{c}_{33} w_{,2} - \frac{h}{2}\bar{c}_{34} v_{,2}) \delta w + (\bar{c}_{14} u_2 + \frac{h}{2}\bar{c}_{24} a_2 - \frac{h}{2}\bar{c}_{34} w_{,2} - \frac{h}{2}\bar{c}_{44} v_{,2}) \delta v]_0^{l_2} \, dx_1, \tag{5}$$

$$\bar{c}_{12} = t_3(\bar{\gamma}_1 - \bar{\gamma}_2), \quad \bar{c}_{13} = \bar{\gamma}_1 t_1 - \bar{\gamma}_2 t_2, \quad \bar{c}_{23} = t_3(\bar{\gamma}_1 t_1 + \bar{\gamma}_2 t_2), \quad \bar{c}_{22} = t_3^2(\bar{\gamma}_1 + \bar{\gamma}_2 + \frac{1}{3}\bar{\gamma}_3),$$

$$\bar{c}_{33} = \frac{4}{3}(\bar{\gamma}_1 t_1^2 + \bar{\gamma}_2 t_2^2), \quad \bar{c}_{14} = t_3^{-1}(\bar{c}_{23} + \frac{1}{2}\bar{c}_{22}), \quad \bar{c}_{24} = t_3(\bar{c}_{13} + \frac{1}{2}\bar{c}_{12}), \quad \bar{c}_{34} = \frac{4}{3}(\bar{\gamma}_1 t_1^2 - \bar{\gamma}_2 t_2^2) + \frac{1}{2} t_3 \bar{c}_{13},$$

$$\bar{c}_{44} = \frac{1}{4} t_3^2(\bar{\gamma}_1 + \bar{\gamma}_2 + \frac{1}{5}\bar{\gamma}_3) + \bar{c}_{23} + \bar{c}_{33}, \quad \bar{c}_{55} = \bar{\gamma}_1 t_1 (12 t_3^2 + 6 t_3 t_1 + t_1^2) + \bar{\gamma}_2 t_2 (12 t_3^2 + 6 t_3 t_2 + t_2^2) + 4 \bar{\gamma}_3 t_3^3.$$

Equating to zero the sum of variations (4) and (5) and collecting together the expressions in front of the variations of the independent variables $\delta u_i$, $\delta a_i$, $\delta w$, $\delta v$, we obtain the system of equations of dynamic stability of a three-layer plate:

$$\begin{gathered}
N_{11,1} + N_{21,2} = \rho h \frac{\partial^2}{\partial t^2}(u_1 + \frac{h}{2}\bar{c}_{12} a_1 - \frac{h}{2}\bar{c}_{13} w_{,1} - \frac{h}{2}\bar{c}_{14} v_{,1}), \\
N_{12,1} + N_{22,2} = \rho h \frac{\partial^2}{\partial t^2}(u_2 + \frac{h}{2}\bar{c}_{12} a_2 - \frac{h}{2}\bar{c}_{13} w_{,2} - \frac{h}{2}\bar{c}_{14} v_{,2}), \\
H_{11,1} + H_{21,2} - Q_1^3 = \frac{\rho h^2}{2} \frac{\partial^2}{\partial t^2}(\bar{c}_{12} u_1 + \frac{h}{2}\bar{c}_{22} a_1 - \frac{h}{2}\bar{c}_{23} w_{,1} - \frac{h}{2}\bar{c}_{24} v_{,1}), \\
H_{12,1} + H_{22,2} - Q_2^3 = \frac{\rho h^2}{2} \frac{\partial^2}{\partial t^2}(\bar{c}_{12} u_2 + \frac{h}{2}\bar{c}_{22} a_2 - \frac{h}{2}\bar{c}_{23} w_{,2} - \frac{h}{2}\bar{c}_{24} v_{,2}), \\
\sum_{i=1}^2 \Big[ \sum_{j=1}^2 (M_{ij,ij} - T_{ij} R_{,ij}) + Q_{i,i}^3 \Big] = \frac{\rho h^2}{2} \frac{\partial^2}{\partial t^2} \sum_{i=1}^2 [\bar{c}_{13} u_{i,i} + \frac{h}{2}\bar{c}_{23} a_{i,i} \\
- \frac{h}{2}\bar{c}_{33} w_{,ii} - \frac{h}{2}\bar{c}_{34} v_{,ii} + h^{-1} w + (t_3 h)^{-1}(\bar{c}_{12} + \bar{c}_{13}) v],
\end{gathered} \tag{6}$$

$$\sum_{i=1}^{2}\sum_{j=1}^{2}(L_{ij,ij}-T_{ij}S_{,ij})-\frac{1}{c}Q_3^3=\frac{\rho h^2}{2}\frac{\partial^2}{\partial t^2}\sum_{i=1}^{2}[\bar c_{14}u_{i,i}+\frac{h}{2}\bar c_{24}a_{i,i}$$
$$-\frac{h}{2}\bar c_{34}w_{,ii}-\frac{h}{2}\bar c_{44}v_{,ii}+\frac{1}{t_3h}(\bar c_{12}+\bar c_{13})w+\frac{1}{3t_3^2h}\bar c_{55}v].$$

The system can be written in terms of displacements. Let us introduce the potentials

$$u_1=\frac{h}{2}(u_{,1}+f_{,2}),\quad u_2=\frac{h}{2}(u_{,2}-f_{,1}),\quad a_1=a_{,1}+\varphi_{,2},\quad a_2=a_{,2}-\varphi_{,1}.$$

The equilibrium equations (6) become

$$\frac{Eh^2}{2(1-\nu^2)}\{\Delta[(u+c_{12}a-c_{13}w-c_{14}v)_{,1}+\frac{1-\nu}{2}(f+c_{12}\varphi)_{,2}]\}=\frac{\rho h^2}{2}\frac{\partial^2}{\partial t^2}[(u+\bar c_{12}a-\bar c_{13}w-\bar c_{14}v)_{,1}+(f+\bar c_{12}\varphi)_{,2}],$$

$$\frac{Eh^2}{2(1-\nu^2)}\{\Delta[(u+c_{12}a-c_{13}w-c_{14}v)_{,2}-\frac{1-\nu}{2}(f+c_{12}\varphi)_{,1}]\}=\frac{\rho h^2}{2}\frac{\partial^2}{\partial t^2}[(u+\bar c_{12}a-\bar c_{13}w-\bar c_{14}v)_{,2}-(f+\bar c_{12}\varphi)_{,1}],$$

$$D_1[\Delta(c_{12}u+c_{22}a-c_{23}w-c_{24}v)_{,1}+\frac{1-\nu}{2}\Delta(c_{12}f+c_{22}\varphi)_{,2}]-Ght_3[(a+w)_{,1}+\varphi_{,2}]$$
$$=\frac{\rho h^3}{4}\frac{\partial^2}{\partial t^2}[(\bar c_{12}u+\bar c_{22}a-\bar c_{23}w-\bar c_{24}v)_{,1}+(\bar c_{12}f+\bar c_{22}\varphi)_{,2}],$$

$$D_1[\Delta(c_{12}u+c_{22}a-c_{23}w-c_{24}v)_{,2}-\frac{1-\nu}{2}\Delta(c_{12}f+c_{22}\varphi)-Ght_3[(a+w)_{,2}-\varphi_{,1}] \tag{7}$$
$$=\frac{\rho h^3}{4}\frac{\partial^2}{\partial t^2}[(\bar c_{12}u+\bar c_{22}a-\bar c_{23}w-\bar c_{24}v)_{,2}-(\bar c_{12}f+\bar c_{22}\varphi)_{,1}],$$

$$D_1\Delta\Delta(c_{13}u+c_{23}a-c_{33}w-c_{34}v)+Ght_3\Delta(a+w)-TR$$
$$=\frac{\rho h^3}{4}\frac{\partial^2}{\partial t^2}\{\bar c_{13}\Delta u+\bar c_{23}\Delta a-(\bar c_{33}\Delta-\frac{4}{h^2})w-[\bar c_{34}\Delta-\frac{t_3}{h^2}(\bar c_{12}+\bar c_{13})]v\},$$

$$D_1\Delta\Delta(c_{14}u+c_{24}a-c_{34}w-c_{44}v)-TS-\frac{4E_0}{ht_3}v$$
$$=\frac{\rho h^3}{4}\frac{\partial^2}{\partial t^2}\{\bar c_{14}\Delta u+\bar c_{24}\Delta a-[\bar c_{34}\Delta-\frac{4}{h^2t_3}(\bar c_{13}+\bar c_{12})]w-[\bar c_{44}\Delta-\frac{4}{3(ht_3)^2}\bar c_{55}]v\},$$

$$\Delta(\ )=(\ )_{,11}+(\ )_{,22}.$$

Parameters $c_{ij}$ are calculated by the same formulas as $\bar c_{ij}$, but with the difference that in the latter $\bar\gamma_k$ should be replaved by $\gamma_k$; $D_1=Eh^3/(4(1-\nu^2))$.

The boundary conditions obtained by equating to zero the displacement variations coefficients are formulated in [1]. There is a distinction only for the edge free from fixation and external load. For instance, the boundary conditions from [1, case *c*] will be as follows ($x_1=x_1^0$):

$$H_{11}^0=0,\quad M_{11}^0=0,\quad L_{11}^0=0,\quad H_{12}^0=0,$$

$$M_{11,1}^0+2M_{12,2}^0+Q_1^3=\frac{\rho h^3}{4}\frac{\partial^2}{\partial t^2}[(\bar c_{13}u+\bar c_{23}a-\bar c_{33}w-\bar c_{34}v)_{,1}+(\bar c_{13}f+\bar c_{23}\varphi)_{,2}],$$

$$L_{11,1}^0+2L_{12,2}^0=\frac{\rho h^3}{4}\frac{\partial^2}{\partial t^2}[(\bar c_{14}u+\bar c_{24}a-\bar c_{34}w-\bar c_{44}v)_{,1}+(\bar c_{14}f+\bar c_{24}\varphi)_{,2}],$$

i.e., there are dynamical terms in the boundary conditions. Equations (7) are significantly simplified for a plate of symmetric structure, i.e., when $h_1=h_2$, $\rho_1=\rho_2$, $E_1=E_2$, then for coefficients $c_{ij}$ and $\bar c_{ij}$ the following equalities are valid $c_{12}=\bar c_{12}=c_{13}=\bar c_{13}=c_{24}=\bar c_{24}=c_{34}=\bar c_{34}=0$.

Equations (7) in this case will take the form

$$\frac{Eh^2}{2(1-\nu^2)}\Delta\Big[(u-c_{14}v)_{,1}+\frac{1-\nu}{2}f_{,2}\Big]=\frac{\rho h^2}{2}\frac{\partial^2}{\partial t^2}[(u-\bar{c}_{14}v)_{,1}+f_{,2}],$$

$$\frac{Eh^2}{2(1-\nu^2)}\Delta\Big[(u-c_{14}v)_{,2}-\frac{1-\nu}{2}f_{,1}\Big]=\frac{\rho h^2}{2}\frac{\partial^2}{\partial t^2}[(u-\bar{c}_{14}v)_{,2}-f_{,1}],$$

$$D_1\Delta\Big[(c_{22}a-c_{23}w)_{,1}+\frac{1-\nu}{2}c_{22}\varphi_{,2}]-Ght_3[(a+w)_{,1}+\varphi_{,2}\Big]=\frac{\rho h^3}{4}\frac{\partial^2}{\partial t^2}[(\bar{c}_{22}a-\bar{c}_{23}w)_{,1}+\bar{c}_{22}\varphi_{,2}],$$

$$D_1\Delta\Big[(c_{22}a-c_{23}w)_{,2}-\frac{1-\nu}{2}c_{22}\varphi_{,1}]+Ght_3[(a+w)_{,2}-\varphi_{,1}\Big]=\frac{\rho h^3}{4}\frac{\partial^2}{\partial t^2}[(\bar{c}_{22}a-\bar{c}_{23}w)_{,2}-\bar{c}_{22}\varphi_{,1}],$$

$$D_1\Delta\Delta(c_{23}a-c_{33}w)+Ght_3\Delta(a+w)-TR=\frac{\rho h^3}{4}\frac{\partial^2}{\partial t^2}\Big[\bar{c}_{23}\Delta a-(\bar{c}_{33}\Delta-\frac{4}{h^2})w\Big],$$

$$D_1\Delta\Delta(c_{14}u-c_{44}v)-TS-\frac{4E_0}{ht_3}v=\frac{\rho h^3}{4}\frac{\partial^2}{\partial t^2}\Big[\bar{c}_{14}\Delta u-(\bar{c}_{44}\Delta-\frac{4}{3t_3^2h^2}\bar{c}_{55})v\Big],$$

$$R=w,\quad S=(\gamma_1+\gamma_2+\frac{1}{3}\gamma_3)w.$$

For plates of symmetric structure the system of equations (7) divides into two systems, one of them describes the local loss of stability and the other, the total loss. Similar simplifications occur also in the boundary conditions. From the third and fourth equations of (7) one can get one equation to define the function $\varphi$. The latter equation is related to the other equations by means of boundary conditions. It was assumed in [1] that the influence of the function $\varphi$ on the static loss of stability is insignificant, and in some cases without great error we may assume $\varphi\equiv 0$; we suspect that the same can be assumed in studing the dynamic stability. It was not hard to get, from the earlier derived equations and boundary conditions, equations and boundary conditions for the problem of the dynamic stability of a three-layer rod.

## REFERENCES


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